A Level Mathematics
Year 11 – 12
Transition Booklet 2025

Name: _____

Compulsory Work

- Complete all tasks on the following page, using the checklist to monitor your progress.
- Use the video links to complete the notes and examples.
- All work should be marked in a different colour and corrections made.
- Full working must be shown for each question, structured clearly and well organised.
- If you have any questions email for help at ajr@churchdownschool.com

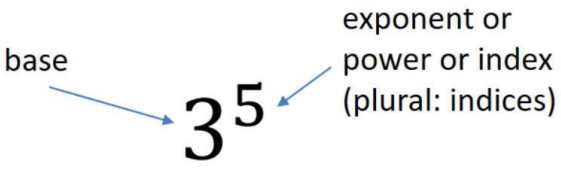
Checklist and Support

	Support Video	Answers	Completed
Exercise 1A			☐ Notes and examples ☐ Exercise
Exercise 1D			☐ Notes and examples ☐ Exercise
Exercise 1E			☐ Notes and examples ☐ Exercise
Exercise 1F			☐ Notes and examples ☐ Exercise
Mixed Exercise			☐ Exercise
Exam Questions			☐ Exam questions

Wider reading

- https://my.integralmaths.org/integral/self_reg/self_reg_students.php
- Registration is free, but you will need to sign up.

Exercise 1A – Notes and Examples



$$a^m \times a^n = a^{m+n}$$
$$a^m \div a^n = a^{m-n}$$
$$(a^m)^n = a^{mn}$$
$$(ab)^n = a^n b^n$$

Simplify $(a^3)^2 \times 2a^2$

Simplify $(4x^3y)^3$

Simplify $2x^2(3+5x) - x(4-x^2)$

Simplify $\frac{x^3-2x}{3x^2}$

Pro Tip: A common student error is to get the sign wrong of $+x^3$

Pro Tip: While $\frac{a+b}{c}$ can be split into $\frac{a}{c}+\frac{b}{c}$, a common student error is to think that $\frac{a}{b+c}=\frac{a}{b}+\frac{a}{c}$

Your Turn

1 Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

2 Simplify $\frac{2x+x^5}{4x^3}$

3 Expand and simplify
$2x(3-x^2) - 4x^3(3-x)$

4 Simplify $2^x \times 3^x$

Note: This is using $(ab)^n = a^n b^n$ law backwards.

Exercise 1A – Questions and Extension

1 Simplify these expressions:

a $x^3 \times x^4$ **b** $2x^3 \times 3x^2$ **c** $\frac{k^3}{k^2}$

d $\frac{4p^3}{2p}$ **e** $\frac{3x^3}{3x^2}$ **f** $(y^2)^5$

g $10x^5 \div 2x^3$ **h** $(p^3)^2 \div p^4$ **i** $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$ **k** $2a^4 \times 3a^5$ **l** $\frac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$ **n** $3x^3 \times 2x^2 \times 4x^6$ **o** $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$ **q** $2a^3 \div 3a^2 \times 6a^5$ **r** $3a^4 \times 2a^5 \times a^3$

3 Simplify these fractions:

a $\frac{6x^4 + 10x^6}{2x}$ **b** $\frac{3x^5 - x^7}{x}$ **c** $\frac{2x^4 - 4x^2}{4x}$

d $\frac{8x^3 + 5x}{2x}$ **e** $\frac{7x^7 + 5x^2}{5x}$ **f** $\frac{9x^5 - 5x^3}{3x}$

Extension

1 *[MAT 2006 1A]*

Which of the following numbers is largest?

- $\left((2^3)^2\right)^3$
- $(2^3)^{(2^3)}$
- $2^{\left((3^2)^3\right)}$
- $2^{\left(3^{(2^3)}\right)}$

2 *[MAT 2012 1B]*

Let $N = 2^k \times 4^m \times 8^n$ where k, m, n are positive whole numbers.

Then N will definitely be a square number whenever:

- k is even;
- $k + n$ is odd;
- k is odd but $m + n$ is even;
- $k + n$ is even.

Exercise 1D – Notes and Examples

$$a^0 = 1$$
$$a^{\frac{1}{m}} = \sqrt[m]{a}$$
$$a^{\frac{n}{m}} = (\sqrt[m]{a})^n$$
$$a^{-m} = \frac{1}{a^m}$$

Pro Note: $\sqrt{9}$ only means the positive square root of 9, i.e. 3 not -3. Otherwise, what would be the point of the $\pm$ in the quadratic formula before the $\sqrt{b^2 - 4ac}$?

Prove that $x^{\frac{1}{2}} = \sqrt{x}$

Evaluate $27^{-\frac{1}{3}}$

Evaluate $32^{\frac{2}{5}}$

Simplify $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

Evaluate $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form kb^n where k, n are constants.

Exercise 1D – Questions and Extension

1 Simplify:

a $x^3 \div x^{-2}$ **b** $x^5 \div x^7$ **c** $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$ **e** $(x^3)^{\frac{5}{3}}$ **f** $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$ **h** $5x^{\frac{7}{3}} \div x^{\frac{2}{5}}$ **i** $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$ **k** $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$ **l** $\dfrac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate:

a $25^{\frac{1}{2}}$ **b** $81^{\frac{3}{2}}$ **c** $27^{\frac{1}{3}}$

d 4^{-2} **e** $9^{-\frac{1}{2}}$ **f** $(-5)^{-3}$

g $\left(\frac{3}{4}\right)^0$ **h** $1296^{\frac{3}{4}}$ **i** $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

j $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ **k** $\left(\frac{6}{5}\right)^{-1}$ **l** $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$ **b** $\dfrac{5x^3 - 2x^2}{x^5}$ **c** $(125x^{12})^{\frac{1}{3}}$ **d** $\dfrac{x + 4x^3}{x^3}$

e $\dfrac{2x + x^2}{x^4}$ **f** $\left(\dfrac{4}{9}x^4\right)^{\frac{3}{2}}$ **g** $\dfrac{9x^2 - 15x^5}{3x^3}$ **h** $\dfrac{5x + 3x^2}{15x^3}$

Extension

[MAT 2007 1A]

Let r and s be integers. Then

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

is an integer if

- $r + s \leq 0$
- $s \leq 0$
- $r \leq 0$
- $r \geq s$

Hint:

Exercise 1E – Questions and Extension

Recap:

A surd is a root of a number that does not simplify to a rational number.

Laws:

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$
$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Note: A *rational* number is any which can be expressed as $\frac{a}{b}$ where a, b are integers. $\frac{2}{3}$ and $\frac{4}{1} = 4$ are rational numbers, but π and $\sqrt{2}$ are not.

$\sqrt{3} \times 2$

$\sqrt{12} + \sqrt{27}$

$3\sqrt{5} \times 2\sqrt{5}$

$(\sqrt{8} + 1)(\sqrt{2} - 3)$

$\sqrt{8}$

Exercise 1E – Questions and Extension

1 Do not use your calculator for this exercise. Simplify:

a $\sqrt{28}$	**b** $\sqrt{72}$	**c** $\sqrt{50}$
d $\sqrt{32}$	**e** $\sqrt{90}$	**f** $\frac{\sqrt{12}}{2}$
g $\frac{\sqrt{27}}{3}$	**h** $\sqrt{20}+\sqrt{80}$	**i** $\sqrt{200}+\sqrt{18}-\sqrt{72}$
j $\sqrt{175}+\sqrt{63}+2\sqrt{28}$	**k** $\sqrt{28}-2\sqrt{63}+\sqrt{7}$	**l** $\sqrt{80}-2\sqrt{20}+3\sqrt{45}$
m $3\sqrt{80}-2\sqrt{20}+5\sqrt{45}$	**n** $\frac{\sqrt{44}}{\sqrt{11}}$	**o** $\sqrt{12}+3\sqrt{48}+\sqrt{75}$

2 Expand and simplify if possible:

a $\sqrt{3}(2+\sqrt{3})$	**b** $\sqrt{5}(3-\sqrt{3})$	**c** $\sqrt{2}(4-\sqrt{5})$
d $(2-\sqrt{2})(3+\sqrt{5})$	**e** $(2-\sqrt{3})(3-\sqrt{7})$	**f** $(4+\sqrt{5})(2+\sqrt{5})$
g $(5-\sqrt{3})(1-\sqrt{3})$	**h** $(4+\sqrt{3})(2-\sqrt{3})$	**i** $(7-\sqrt{11})(2+\sqrt{11})$

3 Simplify $\sqrt{75}-\sqrt{12}$ giving your answer in the form $a\sqrt{3}$, where a is an integer.

Extension

[SMC 2014 Q24] Which of the following is smallest?

- $10-3\sqrt{11}$
- $8-3\sqrt{7}$
- $5-2\sqrt{6}$
- $9-4\sqrt{5}$
- $7-4\sqrt{3}$

Hint:

Exercise 1F – Notes and Examples

Here's a surd.
What could we multiply it by such that it's no longer an irrational number?

$$\sqrt{5}$$

In this fraction, the denominator is irrational.
'**Rationalising the denominator**' means making the denominator a rational number.
What could we multiply this fraction by to both rationalise the denominator, but <u>leave the value of the fraction unchanged?</u>

$$\frac{1}{\sqrt{2}}$$

Side Note: There's two reasons why we might want to do this:

1. For aesthetic reasons, it makes more sense to say "half of root 2" rather than "one root two-th of 1". It's nice to divide by something whole!
2. It makes it easier for us to add expressions involving surds.

$$\frac{3}{\sqrt{2}} =$$

$$\frac{6}{\sqrt{3}} =$$

$$\frac{7}{\sqrt{7}} =$$

$$\frac{15}{\sqrt{5}} + \sqrt{5} =$$

Test Your Understanding:

$$\frac{12}{\sqrt{3}} =$$

$$\frac{2}{\sqrt{6}} =$$

$$\frac{4\sqrt{2}}{\sqrt{8}} =$$

More Complex Denominators

$$\frac{1}{\sqrt{2}+1}$$

We basically use the same expression but with the sign reversed (this is known as the *conjugate*). That way, we obtain the difference of two squares. Since $(a+b)(a-b)=a^2-b^2$, any surds will be squared and thus we'll end up with no surds in the denominator.

$$\frac{3}{\sqrt{6}-2}$$

You can explicitly expand out $(\sqrt{6}-2)(\sqrt{6}+2)$ in the denominator, but remember that $(a-b)(a+b)=a^2-b^2$ so we can mentally obtain $6-4=2$
Just remember: 'difference of two squares'!

$$\frac{4}{\sqrt{3}+1}$$

$$\frac{3\sqrt{2}+4}{5\sqrt{2}-7}$$

Your Turn

Rationalise the denominator and simplify

$$\frac{4}{\sqrt{5}-2}$$

Rationalise the denominator and simplify

$$\frac{2\sqrt{3}-1}{3\sqrt{3}+1}$$

AQA IGCSE FM June 2013 Paper 1

Solve $y(\sqrt{3}-1)=8$

Give your answer in the form $a+b\sqrt{3}$ where a and b are integers.

Exercise 1F – Exercise

1 Rationalise the denominator and simplify the following:

a $\frac{1}{\sqrt{5}+2}=$

b $\frac{\sqrt{3}}{\sqrt{3}-1}=$

c $\frac{\sqrt{5}+1}{\sqrt{5}-2}=$

d $\frac{2\sqrt{3}-1}{3\sqrt{3}+4}=$

e $\frac{5\sqrt{5}-2}{2\sqrt{5}-3}=$

2 Expand and simplify:

$$(\sqrt{5}+3)(\sqrt{5}-2)(\sqrt{5}+1)=$$

3 Rationalise the denominator, giving your answer in the form $a+b\sqrt{3}$.

$$\frac{3\sqrt{3}+7}{3\sqrt{3}-5}=$$

4 Solve $x(4-\sqrt{6})=10$ giving your answer in the form $a+b\sqrt{6}$.

5 Solve $y(1+\sqrt{2})-\sqrt{2}=3$

$$y=\frac{3+\sqrt{2}}{1+\sqrt{2}}=$$

6 Simplify:

$$\frac{\sqrt{a+1}-\sqrt{a}}{\sqrt{a+1}+\sqrt{a}}=$$

Mixed Exercise

(E) **15** Given that $y = \frac{1}{64}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(1 mark)**

b $4y^{-1}$ **(1 mark)**

(E/P) **16** Show that $\frac{5}{\sqrt{75} - \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. **(5 marks)**

(E) **17** Expand and simplify $(\sqrt{11} - 5)(5 - \sqrt{11})$. **(2 marks)**

(E) **18** Factorise completely $x - 64x^3$. **(3 marks)**

(E/P) **19** Express 27^{2x+1} in the form 3^y, stating y in terms of x. **(2 marks)**

(E/P) **20** Solve the equation $8 + x\sqrt{12} = \frac{8x}{\sqrt{3}}$

Give your answer in the form $a\sqrt{b}$ where a and b are integers. **(4 marks)**

(P) **21** A rectangle has a length of $(1 + \sqrt{3})$ cm and area of $\sqrt{12}$ cm^2.
Calculate the width of the rectangle in cm.
Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.

(E) **22** Show that $\frac{(2 - \sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$. **(2 marks)**

(E/P) **23** Given that $243\sqrt{3} = 3^a$, find the value of a. **(3 marks)**

(E/P) **24** Given that $\frac{4x^3 + x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$, write down the value of a and the value of b. **(2 marks)**

Exam Questions

Question 1

Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$ **(2 marks)**

Question 2

Given that $32\sqrt{2} = 2^a$, find the value of a . **(3 marks)**

Question 3

Simplify fully $x^2\left(4x^{-\frac{1}{2}}\right)^2$ **(2 marks)**

Question 4

$f(x) = \frac{(3-4\sqrt{x})^2}{\sqrt{x}}$, $x > 0$

Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found. **(3 marks)**

Question 5

Simplify

$$\frac{7+\sqrt{5}}{\sqrt{5}-1}$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are integers. **(2 marks)**

Question 6

Solve the equation

$$10 + x\sqrt{8} = \frac{6x}{\sqrt{2}}$$

Give your answer in the form $a\sqrt{b}$ where a and b are integers. **(4 marks)**

Answers

Question 1

$\left\{\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}\right\} = \left(\frac{4}{25x^4}\right)^{\frac{1}{2}}$ or $\left(\frac{5x^2}{2}\right)^{-1}$ or $\frac{1}{\left(\frac{25x^4}{4}\right)^{\frac{1}{2}}}$ — *See notes below* — M1

$= \frac{2}{5x^2}$ or $\frac{2}{5}x^{-2}$ — *See notes for other alternatives* — A1

Question 2

$32 = 2^5$ or $2048 = 2^{11}$, $\sqrt{2} = 2^{\frac{1}{2}}$ or $\sqrt{2048} = (2048)^{\frac{1}{2}}$ — B1, B1

$a = \frac{11}{2}$ $\left(\text{or } 5\frac{1}{2} \text{ or } 5.5\right)$ — B1

[3]

Question 3

$(4x^{-\frac{1}{2}})^2 = 16x^{-\frac{2}{2}}$ or $\frac{16}{x}$ or equivalent — M1

$x^2(4x^{-\frac{1}{2}})^2 = 16x$ — A1

Question 4

$\left[(3-4\sqrt{x})^2 =\right] 9 - 12\sqrt{x} - 12\sqrt{x} + (-4)^2 x$ — M1

$9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$ — A1, A1 (3)

Question 5

$\frac{7+\sqrt{5}}{\sqrt{5}-1} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)}$	Multiplies top and bottom by a correct expression. This statement is sufficient.	M1
(Allow to multiply top and bottom by $k(\sqrt{5}+1)$)		
$= \frac{\ldots}{4}$	Obtains a denominator of 4 or sight of $(\sqrt{5}-1)(\sqrt{5}+1) = 4$	A1cso

Question 6

$x\sqrt{8} + 10 = \frac{6x}{\sqrt{2}}$

$\times\sqrt{2} \Rightarrow x\sqrt{16} + 10\sqrt{2} = 6x$ — M1,A1

$4x + 10\sqrt{2} = 6x \Rightarrow 2x = 10\sqrt{2}$

$x = 5\sqrt{2}$ or $a = 5$ and $b = 2$ — M1A1