

A Level Mathematics

Year 11 – 12

Transition Booklet 2026



Name: _____

Compulsory Work

- “Introduce” tasks fully completed for each topic. Please use the checklist on the next page to monitor your progress and ensure they are completed.
- Full working must be shown for each question.
- If you have access to a Sparx account, you can use the Sparx video codes to help with any topics you find challenging. If you do not, please use the videos at <https://sites.google.com/view/tlmaths/home/gcse-to-a-level-maths-bridging-the-gap> to help.
- If you have any questions please feel free email for help at ajr@churchdownschool.com

Optional Work

- Complete the “Strengthen” and “Deepen” tasks.
- These are highly recommended and will provide a solid foundation on which to start the course.
- Sign up and complete the Transition to A Level course at <https://amsp.org.uk/resource/supporting-students-transition-to-level-3-study>

Checklist and Support

	Support Video	Answers	Completed
Exercise 1A			☐ Notes and examples ☐ Exercise
Exercise 1D			☐ Notes and examples ☐ Exercise
Exercise 1E			☐ Notes and examples ☐ Exercise
Exercise 1F			☐ Notes and examples ☐ Exercise
Mixed Exercise			☐ Exercise
Exam Questions			☐ Exam questions

Wider reading

- https://my.integralmaths.org/integral/self_reg/self_reg_students.php
- Registration is free, but you will need to sign up.

Exercise 1A – Notes and Examples

base $\rightarrow$ 3^5 $\leftarrow$ exponent or power or index (plural: indices)



$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(ab)^n = a^n b^n$$

Simplify $(a^3)^2 \times 2a^2$

Simplify $(4x^3y)^3$

Simplify $2x^2(3 + 5x) - x(4 - x^2)$

Simplify $\frac{x^3 - 2x}{3x^2}$

Pro Tip: A common student error is to get the sign wrong of $+x^3$

Pro Tip: While $\frac{a+b}{c}$ can be split into $\frac{a}{c} + \frac{b}{c}$, a common student error is to think that $\frac{a}{b+c} = \frac{a}{b} + \frac{a}{c}$

Your Turn

1 Simplify $\left(\frac{2a^5}{a^2}\right)^2 \times 3a$

2 Simplify $\frac{2x+x^5}{4x^3}$

3 Expand and simplify $2x(3 - x^2) - 4x^3(3 - x)$

4 Simplify $2^x \times 3^x$

Note: This is using $(ab)^n = a^n b^n$ law backwards.

Exercise 1A – Questions and Extension

1 Simplify these expressions:

a $x^3 \times x^4$

d $\frac{4p^3}{2p}$

g $10x^5 \div 2x^3$

j $8p^4 \div 4p^3$

m $9x^2 \times 3(x^2)^3$

p $(4y^3)^3 \div 2y^3$

b $2x^3 \times 3x^2$

e $\frac{3x^3}{3x^2}$

h $(p^3)^2 \div p^4$

k $2a^4 \times 3a^5$

n $3x^3 \times 2x^2 \times 4x^6$

q $2a^3 \div 3a^2 \times 6a^5$

c $\frac{k^3}{k^2}$

f $(y^2)^5$

i $(2a^3)^2 \div 2a^3$

l $\frac{21a^3b^7}{7ab^4}$

o $7a^4 \times (3a^4)^2$

r $3a^4 \times 2a^5 \times a^3$

3 Simplify these fractions:

a $\frac{6x^4 + 10x^6}{2x}$

d $\frac{8x^3 + 5x}{2x}$

b $\frac{3x^5 - x^7}{x}$

e $\frac{7x^7 + 5x^2}{5x}$

c $\frac{2x^4 - 4x^2}{4x}$

f $\frac{9x^5 - 5x^3}{3x}$

Extension

1 [MAT 2006 1A]

Which of the following numbers is largest?

- ☐ $\left((2^3)^2\right)^3$
- ☐ $(2^3)^{(2^3)}$
- ☐ $2^{((3^2)^3)}$
- ☐ $2^{(3^{(2^3)})}$

2 [MAT 2012 1B]

Let $N = 2^k \times 4^m \times 8^n$ where k, m, n are positive whole numbers.

Then N will definitely be a square number whenever:

- ☐ k is even;
- ☐ $k + n$ is odd;
- ☐ k is odd but $m + n$ is even;
- ☐ $k + n$ is even.

Exercise 1D – Notes and Examples

$$\begin{aligned}a^0 &= 1 \\ a^{\frac{1}{m}} &= \sqrt[m]{a} \\ a^{\frac{n}{m}} &= (\sqrt[m]{a})^n \\ a^{-m} &= \frac{1}{a^m}\end{aligned}$$

Pro Note: $\sqrt{9}$ only means the positive square root of 9, i.e. 3 not -3.
Otherwise, what would be the point of the $\pm$ in the quadratic formula before the $\sqrt{b^2 - 4ac}$?

Prove that $x^{\frac{1}{2}} = \sqrt{x}$

Evaluate $27^{-\frac{1}{3}}$

Evaluate $32^{\frac{2}{5}}$

Simplify $\left(\frac{1}{9}x^6y\right)^{\frac{1}{2}}$

Evaluate $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

If $b = \frac{1}{9}a^2$, determine $3b^{-2}$ in the form kb^n where k, n are constants.

Exercise 1D – Questions and Extension

1 Simplify:

a $x^3 \div x^{-2}$

b $x^5 \div x^7$

c $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$

e $(x^3)^{\frac{5}{3}}$

f $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$

h $5x^{\frac{7}{3}} \div x^{\frac{2}{3}}$

i $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$

k $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$

l $\frac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate:

a $25^{\frac{1}{2}}$

b $81^{\frac{3}{2}}$

c $27^{\frac{1}{3}}$

d 4^{-2}

e $9^{-\frac{1}{2}}$

f $(-5)^{-3}$

g $(\frac{3}{4})^0$

h $1296^{\frac{3}{4}}$

i $(\frac{25}{16})^{\frac{3}{2}}$

j $(\frac{27}{8})^{\frac{2}{3}}$

k $(\frac{6}{5})^{-1}$

l $(\frac{343}{512})^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$

b $\frac{5x^3 - 2x^2}{x^5}$

c $(125x^{12})^{\frac{1}{3}}$

d $\frac{x + 4x^3}{x^3}$

e $\frac{2x + x^2}{x^4}$

f $(\frac{4}{9}x^4)^{\frac{3}{2}}$

g $\frac{9x^2 - 15x^5}{3x^3}$

h $\frac{5x + 3x^2}{15x^3}$

Extension

[MAT 2007 1A]

Let r and s be integers. Then

$$\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$$

Hint:

is an integer if

- ☐ $r + s \leq 0$
- ☐ $s \leq 0$
- ☐ $r \leq 0$
- ☐ $r \geq s$

Exercise 1E – Questions and Extension

Recap:

A surd is a root of a number that does not simplify to a rational number.

Laws:

$$\begin{aligned}\sqrt{a} \times \sqrt{b} &= \sqrt{ab} \\ \frac{\sqrt{a}}{\sqrt{b}} &= \sqrt{\frac{a}{b}}\end{aligned}$$

Note: A *rational* number is any which can be expressed as $\frac{a}{b}$ where a, b are integers. $\frac{2}{3}$ and $\frac{4}{1} = 4$ are rational numbers, but π and $\sqrt{2}$ are not.

$$\sqrt{3} \times 2$$

$$\sqrt{12} + \sqrt{27}$$

$$3\sqrt{5} \times 2\sqrt{5}$$

$$(\sqrt{8} + 1)(\sqrt{2} - 3)$$

$$\sqrt{8}$$

Exercise 1E – Questions and Extension

1 Do not use your calculator for this exercise. Simplify:

a $\sqrt{28}$

b $\sqrt{72}$

c $\sqrt{50}$

d $\sqrt{32}$

e $\sqrt{90}$

f $\frac{\sqrt{12}}{2}$

g $\frac{\sqrt{27}}{3}$

h $\sqrt{20} + \sqrt{80}$

i $\sqrt{200} + \sqrt{18} - \sqrt{72}$

j $\sqrt{175} + \sqrt{63} + 2\sqrt{28}$

k $\sqrt{28} - 2\sqrt{63} + \sqrt{7}$

l $\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$

m $3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$

n $\frac{\sqrt{44}}{\sqrt{11}}$

o $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

2 Expand and simplify if possible:

a $\sqrt{3}(2 + \sqrt{3})$

b $\sqrt{5}(3 - \sqrt{3})$

c $\sqrt{2}(4 - \sqrt{5})$

d $(2 - \sqrt{2})(3 + \sqrt{5})$

e $(2 - \sqrt{3})(3 - \sqrt{7})$

f $(4 + \sqrt{5})(2 + \sqrt{5})$

g $(5 - \sqrt{3})(1 - \sqrt{3})$

h $(4 + \sqrt{3})(2 - \sqrt{3})$

i $(7 - \sqrt{11})(2 + \sqrt{11})$

3 Simplify $\sqrt{75} - \sqrt{12}$ giving your answer in the form $a\sqrt{3}$, where a is an integer.

Extension

[SMC 2014 Q24] Which of the following is smallest?

- ☐ $10 - 3\sqrt{11}$
- ☐ $8 - 3\sqrt{7}$
- ☐ $5 - 2\sqrt{6}$
- ☐ $9 - 4\sqrt{5}$
- ☐ $7 - 4\sqrt{3}$

Hint:

Exercise 1F – Notes and Examples

Here's a surd.

What could we multiply it by such that it's no longer an irrational number?

$$\sqrt{5}$$

In this fraction, the denominator is irrational.

'Rationalising the denominator' means making the denominator a rational number. What could we multiply this fraction by to both rationalise the denominator, but leave the value of the fraction unchanged?

$$\frac{1}{\sqrt{2}}$$

Side Note: There's two reasons why we might want to do this:

1. For aesthetic reasons, it makes more sense to say "half of root 2" rather than "one root two-th of 1". It's nice to divide by something whole!
2. It makes it easier for us to add expressions involving surds.

$$\frac{3}{\sqrt{2}} =$$

$$\frac{6}{\sqrt{3}} =$$

$$\frac{7}{\sqrt{7}} =$$

$$\frac{15}{\sqrt{5}} + \sqrt{5} =$$

Test Your Understanding:

$$\frac{12}{\sqrt{3}} =$$

$$\frac{2}{\sqrt{6}} =$$

$$\frac{4\sqrt{2}}{\sqrt{8}} =$$

More Complex Denominators

$$\frac{1}{\sqrt{2} + 1}$$

We basically use the same expression but with the sign reversed (this is known as the *conjugate*). That way, we obtain the difference of two squares. Since $(a + b)(a - b) = a^2 - b^2$, any surds will be squared and thus we'll end up with no surds in the denominator.

$$\frac{3}{\sqrt{6} - 2}$$

You can explicitly expand out $(\sqrt{6} - 2)(\sqrt{6} + 2)$ in the denominator, but remember that $(a - b)(a + b) = a^2 - b^2$ so we can mentally obtain $6 - 4 = 2$. Just remember: 'difference of two squares'!

$$\frac{4}{\sqrt{3} + 1}$$

$$\frac{3\sqrt{2} + 4}{5\sqrt{2} - 7}$$

Your Turn

Rationalise the denominator and simplify

$$\frac{4}{\sqrt{5} - 2}$$

Rationalise the denominator and simplify

$$\frac{2\sqrt{3} - 1}{3\sqrt{3} + 1}$$

AQA IGCSE FM June 2013 Paper 1

Solve $y(\sqrt{3} - 1) = 8$

Give your answer in the form $a + b\sqrt{3}$ where a and b are integers.

Exercise 1F – Exercise

- 1 Rationalise the denominator and simplify the following:

a $\frac{1}{\sqrt{5} + 2} =$

b $\frac{\sqrt{3}}{\sqrt{3} - 1} =$

c $\frac{\sqrt{5} + 1}{\sqrt{5} - 2} =$

d $\frac{2\sqrt{3} - 1}{3\sqrt{3} + 4} =$

e $\frac{5\sqrt{5} - 2}{2\sqrt{5} - 3} =$

- 2 Expand and simplify:
 $(\sqrt{5} + 3)(\sqrt{5} - 2)(\sqrt{5} + 1) =$

- 3 Rationalise the denominator, giving your answer in the form $a + b\sqrt{3}$.
 $\frac{3\sqrt{3} + 7}{3\sqrt{3} - 5} =$

- 4 Solve $x(4 - \sqrt{6}) = 10$ giving your answer in the form $a + b\sqrt{6}$.

- 5 Solve $y(1 + \sqrt{2}) - \sqrt{2} = 3$
 $y = \frac{3 + \sqrt{2}}{1 + \sqrt{2}} =$

Simplify:

- 6 $\frac{\sqrt{a+1} - \sqrt{a}}{\sqrt{a+1} + \sqrt{a}} =$

Mixed Exercise

- (E)** 15 Given that $y = \frac{1}{64}x^3$ express each of the following in the form kx^n , where k and n are constants.
- a** $y^{\frac{1}{3}}$ (1 mark)
- b** $4y^{-1}$ (1 mark)
- (E/P)** 16 Show that $\frac{5}{\sqrt{75} - \sqrt{50}}$ can be written in the form $\sqrt{a} + \sqrt{b}$, where a and b are integers. (5 marks)
- (E)** 17 Expand and simplify $(\sqrt{11} - 5)(5 - \sqrt{11})$. (2 marks)
- (E)** 18 Factorise completely $x - 64x^3$. (3 marks)
- (E/P)** 19 Express 27^{2x+1} in the form 3^y , stating y in terms of x . (2 marks)
- (E/P)** 20 Solve the equation $8 + x\sqrt{12} = \frac{8x}{\sqrt{3}}$
Give your answer in the form $a\sqrt{b}$ where a and b are integers. (4 marks)
- (P)** 21 A rectangle has a length of $(1 + \sqrt{3})$ cm and area of $\sqrt{12}$ cm².
Calculate the width of the rectangle in cm.
Express your answer in the form $a + b\sqrt{3}$, where a and b are integers to be found.
- (E)** 22 Show that $\frac{(2 - \sqrt{x})^2}{\sqrt{x}}$ can be written as $4x^{-\frac{1}{2}} - 4 + x^{\frac{1}{2}}$. (2 marks)
- (E/P)** 23 Given that $243\sqrt{3} = 3^a$, find the value of a . (3 marks)
- (E/P)** 24 Given that $\frac{4x^3 + x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $4x^a + x^b$, write down the value of a and the value of b . (2 marks)

Exam Questions

Question 1

Simplify fully $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$ **(2 marks)**

Question 2

Given that $32\sqrt{2} = 2^a$, find the value of a . **(3 marks)**

Question 3

Simplify fully $x^2 \left(4x^{-\frac{1}{2}}\right)^2$ **(2 marks)**

Question 4

$$f(x) = \frac{(3-4\sqrt{x})^2}{\sqrt{x}}, x > 0$$

Show that $f(x) = 9x^{-\frac{1}{2}} + Ax^{\frac{1}{2}} + B$, where A and B are constants to be found. **(3 marks)**

Question 5

Simplify

$$\frac{7 + \sqrt{5}}{\sqrt{5} - 1}$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are integers. **(2 marks)**

Question 6

Solve the equation

$$10 + x\sqrt{8} = \frac{6x}{\sqrt{2}}$$

Give your answer in the form $a\sqrt{b}$ where a and b are integers. **(4 marks)**

Answers

Question 1

M1	See notes below
A1	See notes for other alternatives

Question 2

B1, B1	$32 = 2^5$ or $2048 = 2^{11}$, $\sqrt{2} = 2^{\frac{1}{2}}$ or $\sqrt{2048} = (2048)^{\frac{1}{2}}$
B1	$a = \frac{11}{2}$ or $5\frac{1}{2}$ or 5.5
[3]	

Question 3

M1	$(4x^{-\frac{1}{2}})^{-\frac{2}{3}} = 16x^{-\frac{2}{3}}$ or $\frac{x}{16}$ or equivalent
A1	$x^2(4x^{\frac{1}{2}})^{-\frac{2}{3}} = 16x$

Question 4

M1	$[(3-4\sqrt{x})^2 = 9-12\sqrt{x}-12\sqrt{x}+(-4)^2 x]$ $9x^{-\frac{1}{2}}+16x^{\frac{1}{2}}-24$
A1, A1	(3)

Question 5

M1	Multiples top and bottom by a correct expression. This statement is sufficient.
A1cso	$\frac{7+\sqrt{5}}{7+\sqrt{5}} \times \frac{(\sqrt{5}-1)}{(\sqrt{5}-1)}$ (Allow to multiply top and bottom by $k(\sqrt{5}+1)$) $\frac{4}{\dots}$ Obtains a denominator of 4 or sight of $(\sqrt{5}-1)(\sqrt{5}+1)=4$

Question 6

M1,A1	$\frac{x\sqrt{8}+10}{6x} = 10 + \frac{\sqrt{2}}{6x}$ $\sqrt{2} \times \sqrt{2} = 6x \Rightarrow x\sqrt{16}+10\sqrt{2} = 6x$ $4x+10\sqrt{2} = 6x \Rightarrow 2x = 10\sqrt{2}$ or $a = 5$ and $b = 2$
M1A1	$x = 5\sqrt{2}$